Consider the sequence $3, -24, 192, -1536, 12288, \cdots$.

SCORE: _____ / 13 PTS

$\times(-8)$ $\times(-8)$ $\times(-8)$ $\times(-8)$ GEOMETRIC

[a] Find the formula for the n^{th} term of the sequence.

$$a_n = 3(-8)^{n-1}$$ (2)

[b] Find the formula for the sum of the first n terms of the corresponding series $3-24+192-1536+12288-\cdots$.

$$S_n = \frac{3(1-(-8)^n)}{1-(-8)} = \frac{1}{3}(1-(-8)^n)$$ (2)

[c] Prove that your formula in [b] is correct **using mathematical induction**.

BASIS STEP: $S_1 = 3$

(1) $\frac{1}{3}(1-(-8)^1) = \frac{1}{3}(9) = 3$ ✓

INDUCTIVE STEP: ASSUME $3-24+192-\cdots+3(-8)^{k-1} = \frac{1}{3}(1-(-8)^k)$, (1)

(1) FOR SOME PARTICULAR BUT ARBITRARY INTEGER $k \geq 1$

(1) $3-24+192-\cdots+3(-8)^{k-1}+3(-8)^k$

(1) $= \frac{1}{3}(1-(-8)^k)+3(-8)^k$

(1) $= \frac{1}{3}(1-(-8)^k+9(-8)^k)$

$= \frac{1}{3}(1+8(-8)^k)$

(1) $= \frac{1}{3}(1-(-8)(-8)^k)$

$= \frac{1}{3}(1-(-8)^{k+1})$ (1)

(1) BY MI, $3-24+192-\cdots+3(-8)^{n-1} = \frac{1}{3}(1-(-8)^n)$ FOR ALL INTEGERS $n \geq 1$

Find the coefficient of x^6 in the expansion of $(3x^2 - 5)^{37}$. **SCORE: _____ / 5 PTS**
You may write your final answer in factored form, as shown in lecture.
Your final answer must **NOT** contain ! or $C(n, r)$ (or equivalent) notation.
NOTE: Do NOT use your calculator's ! nor $C(n, r)$ feature.

$$\binom{37}{r}(3x^2)^{37-r}(-5)^r = \binom{37}{r} 3^{37-r}(-5)^r x^{2(37-r)}$$

$2(37-r) = 6$
$r = 34$

$$= \binom{37}{34} 3^{37-34}(-5)^{34} x^6$$

(1)

$$= \frac{37!}{34!\,3!} 3^3 5^{34} x^6$$

(1)

$$= \frac{37 \cdot \cancel{36}^{6} \cdot 35 \cdot \cancel{34!}}{\cancel{34!}\,\cancel{3 \cdot 2 \cdot 1}} 3^3 5^{34} x^6 = 37 \cdot 6 \cdot 35 \cdot 3^3 5^{34} x^6$$

(1) (1) (1)

Expand and simplify $(t^2 - 2\sqrt{t})^4$. **The coefficients in your final answer must be completely simplified.** **SCORE: _____ / 5 PTS**

$$(t^2)^4 + 4(t^2)^3(-2\sqrt{t}) + 6(t^2)^2(-2\sqrt{t})^2 + 4(t^2)(-2\sqrt{t})^3 + (-2\sqrt{t})^4$$

$$= t^8 - 8t^6\sqrt{t} + 24t^5 - 32t^3\sqrt{t} + 16t^2$$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1

$\frac{1}{2}$ POINT EACH

OK IF WROTE $t^{6.5}$ AND $t^{3.5}$ INSTEAD OF $t^6\sqrt{t}$ AND $t^3\sqrt{t}$

FJ & GJ were trying to get more customers for their business, so they opened a Twitter account, and on the first day, **SCORE: _____ / 7 PTS** they got 258 new followers. Every day after that, they got additional new followers, so the total number of followers increased. (Their followers were very loyal, and never stopped following their account.) Because Twitter was losing its appeal, every day, the number of new followers was 7 less than the number of new followers the previous day. One day, FJ & GJ noticed that, for the first time ever, the number of new followers that day was less than 50. ARITHMETIC $d = -7$

[a] How many days had their Twitter account been open when they noticed that the number of new followers that day had dropped below 50 for the first time ?

(2) $258 - 7(n-1) < 50$ (1)

$-7(n-1) < -208$

$n - 1 < 29\frac{5}{7}$

$n < 30\frac{5}{7}$

$n = 31$ (1)

DAY 31 WAS THE FIRST DAY WITH FEWER THAN 50 NEW FOLLOWERS.

[b] How many followers did they have altogether at that time ?

$S_{31} = \frac{31}{2}(2(258) - 7(31-1))$ (2)

$= 4743$ FOLLOWERS

(1)